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Development of the Multiscale Version of the SL-AV Global Atmosphere Model

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Abstract—The global hydrodynamic atmosphere model SL-AV is applied for operational medium-range weather forecast and as a component of the probabilistic long-range forecast system. The review of the previous development of the model is presented and the model features are noted. The existing model versions are described. The unified multi-scale version of the model is developed on the basis of these versions. This version is intended both for numerical weather prediction and for modeling of climate changes. The numerical experiments on climate modeling with the developed multi-scale version are carried out according to the protocol of the international AMIP2 experiment. First results are presented. The possibility of application of the unified version of the SL-AV model for the medium-range weather forecast, and, after some development, for modeling of climate changes is shown.

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1. INTRODUCTION

Numerical modeling of the atmosphere was one of G.I. Marchuk activities. His 90th anniversary is celebrated in 2015. Based on his works presented in the monograph “Numerical Methods in Weather Prediction” (published in the USSR in 1967 and then translated to English by Academic Press in 1974 [6]), A. Robert suggested the semi-implicit method [9] that allows increasing the time step in the atmosphere dynamics models by 3–5 times in comparison with the explicit time integration schemes. This method is still used under different flavors in the majority of atmosphere models. One of the first coupled hydrodynamic models of the atmosphere and ocean was developed [7] under the G.I. Marchuk leadership. G.I. Marchuk’s merits in this area were noted in 2000 by the honorable medal of the European Geophysical Society (currently the European Geophysical Union).

[†] Deceased.

The main result of applying the method of splitting into physical processes proved by G.I. Marchuk is the use of the time step which is the greatest possible according to physical considerations, for each of the processes described in the atmosphere model. The further development of this idea was coupling of the semi-implicit time integration method for the atmosphere dynamics equations [7] with the semi-Lagrangian representation of advection [28] and transition to the two-time-level integration scheme [20]. This approach allows increasing the time integration step by several times in comparison with Eulerian methods.

The global semi-Lagrangian model of the atmosphere general circulation SL-AV (Semi-Lagrangian, based on the Absolute Voriticity equation) is developed [11, 12] using the semi-Lagrangian semi-implicit algorithm. The model is used for operational medium-range numerical weather prediction at the Hydrometeorological Center of Russia and Siberian Hydrometeorological Institute (SibNIGMI) and as a component of the probabilistic long-range forecast system at the Hydrometeorological Center of Russia.

The concept of multiscale (or seamless prediction) atmosphere model appeared in 2005 [21, 27]. It suggests that there are no artificial time limits in the atmosphere dividing mesoscale, synoptic, seasonal and interannual scales. All time scales interact due to atmosphere nonlinearity. Thus, the model of the atmosphere general circulation intended for simulation of any time scale has to reproduce processes of all time scales adequately. This often involves a need to couple the atmosphere model to new models being the Earth system components (the ocean, sea ice, the active land layer, small gas constituents etc.). The leading prediction centers, the European Center for Medium-Range Weather Forecasts, and the UK MetOffice plan to apply such models for medium-range weather prediction in the next years.

The work on development of the unified version for the SL-AV model is presented in this article after the review of the previous development of this global hydrodynamic model of atmosphere. Also, the first results of atmosphere circulation simulation are shown. This version can be used both for numerical weather prediction at different lead times and for modeling of climate changes; thus, it is multiscale.

2. FEATURES OF THE GLOBAL ATMOSPHERE MODEL SL-AV

The original features of the block solving the atmosphere dynamics equations in the SL-AV model are the use of the vertical component of the absolute vorticity and divergence as prognostic variables and application of the fourth order finite-differences on the unstaggered grid to approximate non-advective terms of equations. Achieving the maximum accuracy of the medium-range weather forecast with the minimum computational cost was initially the main objective of developing the model. Therefore, the application of the unstaggered grid in horizontal (or grid A, according to Arakawa [8]) seemed desirable as it allowed avoiding both the calculation of several trajectories for each grid cell in a semi-Lagrangian model, and two-dimensional averaging operators. However, as known from the classical work [8], the processes of inertia and the propagation of gravity waves are badly described at grid A when using the traditional formulation of the equations of the atmosphere processes. Applying the vertical component of the absolute vorticity and the horizontal divergence as prognostic variables allows using the unstaggered grid (sometimes called Z grid) in a finite-difference semi-Lagrangian model providing the same characteristics of the propagation of the gravity waves as for grids B and C, and slightly better properties for inertia-gravity and Rossby waves [23, 25].

An important element of the atmosphere model based on such variables as the vertical component of the absolute vorticity and horizontal divergence is a fast and accurate algorithm to reconstruct the horizontal wind speed components [33]. It was shown in [32] that the shallow water version of the SL-AV model on the sphere using this algorithm reached the error level for the most difficult experiments of the set of the standard test [34] typical of the spectral model of equivalent resolution.

Paper [2] presents the three-dimensional version of the model with the resolution of 1.5° in longitude and latitude, 20 levels in the vertical. Later, the model resolution increased, and there was a need for efficient parallel model implementation. The work on model parallelization is presented in [4]. The current state of the block solving the atmosphere dynamics equations and the new version of this block conserving the air mass locally and globally, are described in [26].

The algorithms for the parameterization of subgrid-scale processes developed under the leadership of J.-F. Geleyn by the mesoscale weather prediction Consortium ALADIN/LACE [17, 18] headed by France, are generally adopted in the SL-AV model along with the original block developed in Russia that solves the atmosphere dynamics equations. Parameterizations of large-scale precipitation [4] and the multilevel soil model [3] developed in Russia are also included into the model. The work on the improvement of the description of subgrid-scale processes is going on, its part is briefly presented in this article.

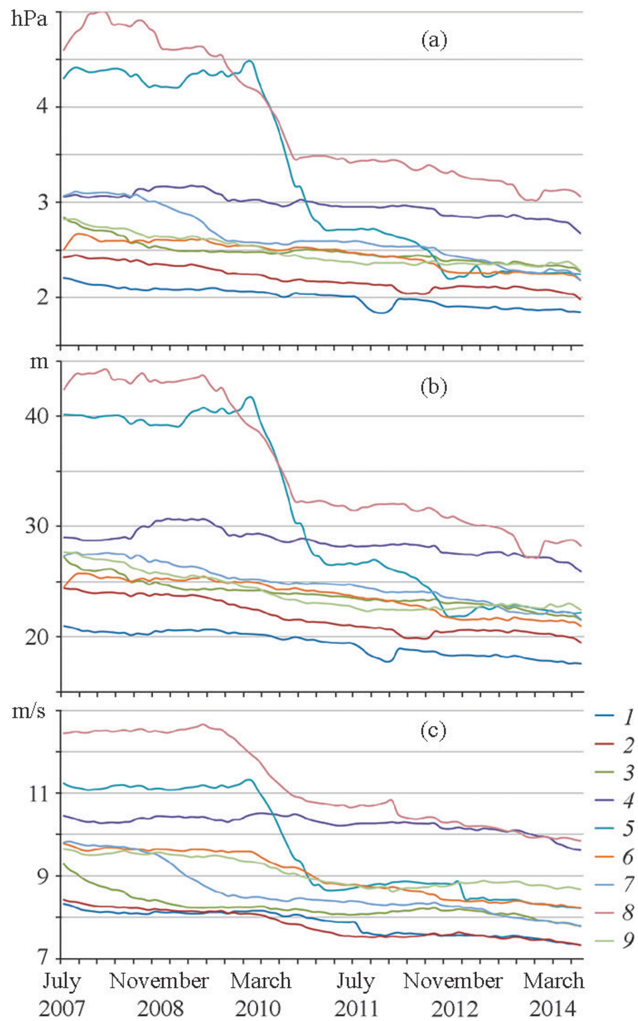

Fig. 1. Average monthly root-mean-squared errors of 72 h forecasts (the moving average over 12 months) for different prediction centers in the extra-tropical part of the Northern Hemisphere with respect to the objective analysis of the relevant center for (a) sea level pressure, (b) isobaric 500 hPa surface height, and (c) the isobaric 250 hPa surface wind module, during 2008–2014 according to the WMO Lead Center for Deterministic Forecast Verification. (*1*) ECMWF; (*2*) UKMO; (*3*) JMA; (*4*) DWD; (*5*) BOM; (*6*) NCEP; (*7*) CMC; (*8*) RUMS; (*9*) Meteo-France.

3. THE EXISTING VERSIONS OF THE SL-AV MODEL

The operational medium-range weather prediction version of the SL-AV model has the horizontal resolution of 0.9 0.72 in longitude and latitude (about 75 km in midlatitudes) and 28 levels in the vertical. The parameterizations generally correspond to the level of the ALADIN model in 2006 with some improvements. Unlike the original parameterization, Kuo-like closure is applied in the deep convection scheme in the SL-AV model in the cases when the temperature at the lower model level is below a certain threshold value, otherwise the closure based on convective available potential energy (CAPE) is applied [30].

At the beginning of 2010 this version was introduced as operational at the Hydrometeorological Center of Russia, and now it is possible to summarize some results. Figure 1a shows the monthly averaged RMS error for the sea level pressure forecast in the extratropical part of the Northern Hemisphere during the period from 2008 to 2014 with respect to the objective analysis field for different weather prediction centers. The similar measure for the 500 hPa height field is presented in Fig. 1b, and Figure 1c shows the wind module RMS error at the level of 250 hPa (following the data of the WMO Lead Center for Deterministic Forecast Verification http://apps.ecmwf.int/wmolcdnv). The introduction of the SL-AV global model as the main numerical method at the beginning of 2010 allowed reducing the gap between

Russia and the leading group of the world prediction centers approximately by twice. The gap concerned forecast errors for such important fields as sea level pressure, 850 hPa surface temperature and 500 hPa surface height.

To increase the accuracy of the near-surface temperature and humidity forecast within operational technology, the assimilation of soil temperature and moisture content was implemented for the first time in Russia [1]. Also for the first time in Russia, the variational objective analysis for near-surface temperature (2D-Var) was implemented in the operational mode [2]. Using this version, the influence of mire parameterization incorporation on the error of the near-surface temperature and humidity forecast is also investigated [35].

In Novosibirsk in SibNIGMI the improved model version is employed operationally with the variable resolution in latitude [31] changing from 30 km in the latitude band of 48 –90 N to 70 km in the Southern Hemisphere. In this model version the resolution in longitude is 0.5625 (62 km at the equator, 31 km at 60 N), the number of vertical levels is 50.

Besides the medium-range weather prediction version of the SL-AV model, the model version focused on probabilistic long-range forecasts is applied at the Hydrometeorological Center of Russia (for time scales from a month to a season). This version has the horizontal resolution of 1.40625 in longitude, 1.125 in latitude and the same number of vertical levels (28). The results for historical seasonal forecasts using this model version are presented in [13] in detail. The dynamic-stochastic scheme for large-scale condensation developed at Institute of Numerical Mathematics of Russian Academy of Sciences [5] is used in this model version. In this case, the condensation in a model grid cell occurs when the value of specific moisture mass fraction is more than the dew-point value, taking into account moisture dispersion in this cell. Dispersion of moisture distribution is set according to observations as a function of temperature and pressure. This scheme is especially useful for the model versions having horizontal resolution about 100 km and more, which are intended for long-range forecasts and climate modeling. This version of the atmosphere model was also coupled to the INM World Ocean model, the results of historical seasonal forecasts are presented in [13].

Since the operational trials of the SL-AV model in 2009, the horizontal and vertical resolution of global models has significantly increased in the world. Therefore, the new version was developed of the SL-AV model for medium-range weather prediction having the horizontal resolution of about 20–25 km and 51 levels in vertical. The resolution in latitude changes from 0.178 in the Northern Hemisphere to 0.24 in the Southern one. Thus, the model resolution in latitude is about 20 km in the middle latitudes of the Northern Hemisphere, and is about 26 km in the Southern one. In this version, more accurate and locally conservative approximations for gradient, divergence and vorticity operators are implemented in the block solving the atmosphere dynamics equations. The increase of accuracy near the poles is most noticeable. Application of such approximations allowed reducing the horizontal diffusion coefficient for divergence that might promote more realistic simulation of kinetic energy spectra. The new model version has more advanced parameterization of subgrid-scale processes. Freely available CLIRAD-SW short-wave radiation parameterization [16, 29] previously tested in [14] and also RRTM free long-wave radiation parameterization [22] are applied. The algorithms developed by the RC-LACE consortium [15] are applied to parameterize the microphysical processes of nonconvective character ([19], parts 1 and 3 of Section 2). The three-dimensional climate distributions of the content of ozone and aerosol depending on a month are also included. This model version is at operational trials at the Hydrometeorological Center of Russia. Preliminary results show some reduction of forecast errors. The detailed description of this version is supposed to be published in a separate article.

4. THE UNIFIED MULTISCALE VERSION OF THE SL-AV MODEL

As stated above, the atmosphere general circulation model aimed at the simulation of a certain time scale has to simulate adequately the processes of all time scales. Besides, the support cost for several different versions of the same model differing by the parameterization sets used, by the nomenclature of prediction fields, etc. is inadequately high. Therefore, the goal was set to develop the global model based on the standardization of several versions of the SL-AV model for seamless prediction at lead times from a day to several years.

The unified version of the model is developed on the basis of the earlier developed versions of the SL-AV global atmosphere model. This version can be used both for numerical weather prediction at different lead times and for modeling of climate changes; thus, it is a multiscale model. The parameterizations of subgrid-

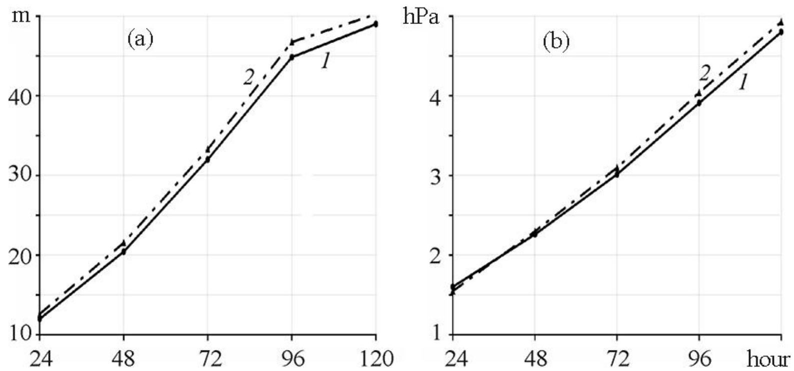

Fig. 2. Root-mean-squared errors of forecasts for 500 hPa isobaric surface height (a) and sea level pressure (b) in the extratropical part of the Northern Hemisphere in January and June, 2014 for the unified (*1*) and operational version (*2*) of the SL-AV model and the lead forecast time from 24 to 120 h (with respect to the Hydrometeorological Center objective analysis).

scale processes generally correspond to the new version of the model intended for a medium-range weather prediction (see above).

At the beginning of the work on this model development the block of parameterization of soil processes developed and used in the INM RAS Earth system model [3] was incorporated into the SL-AV atmosphere model. The INM RAS model describing heat and moisture exchange in the multilayer soil was introduced in the SL-AV atmosphere model instead of the earlier used ISBA parameterization [24] having only two layers in the soil. The INM RAS soil model was supplemented with the advanced description for the dependence of heat conductivity coefficients on water and ice content. Eight soil layers are used. The results of test calculations for the territory of Russia and Asia showed an advantage of the new method describing heat and moisture exchange in the soil as compared with the earlier applied scheme. This advantage concerns the calculations for the period of soil melting (April), both for medium-range and long-range numerical weather forecast. These works are supposed to be covered in a separate article in more detail.

The multiscale version of the model includes the advanced scheme of broadband snow albedo parameterization. In this scheme, the change of this parameter during fresh snow fall occurs smoothly, and also the albedo dependence on the solar zenith angle is considered [10].

The periodic change of the lower boundary conditions and also the output of necessary characteristics according to the protocol requirements of the international experiments AMIP, CHFP, etc., is provided in the climate modeling mode in the program complex of the unified model version (for example, all the components for momentum, heat and moisture fluxes at the surface). The data output necessary for modeling of climate changes was parallelized using MPI technology by data collection onto the master processor. The horizontal resolution of the model in the configuration for the research of climate changes is 0.9 0.72 in longitude and latitude respectively, 28 levels in the vertical (coincides with the resolution of the operational version for the medium-range weather forecast).

For the unified version of the SL-AV model, the mass-conserving version of the dynamic block of the model [26] is developed and verified. It should be noted that the local and global mass conservation of transported substance (air and small gas components) is among the important requirements to the atmosphere models intended for modeling of climate changes.

5. RESULTS OF NUMERICAL EXPERIMENTS

The unified version of the model was validated with medium-range weather forecasts with the lead time of 120 h for all days of January and July, 2014. The model resolution was the same as for climate version. Forecasts started with initial data for 12:00 UTC. Figure 2 shows the comparison of root-mean squared forecast errors in the extratropical part of the Northern Hemisphere for 500 hPa isobaric surface height and sea level pressure for the unified model and the operational version of the SL-AV model at the forecast lead time from 24 to 120 h (with respect to the Hydrometeorological Center objective analysis). One can see that the medium-range forecast error is slightly smaller for the model unified version due to the more accurate parameterizations of subgrid-scale processes and improvements in the dynamical core.

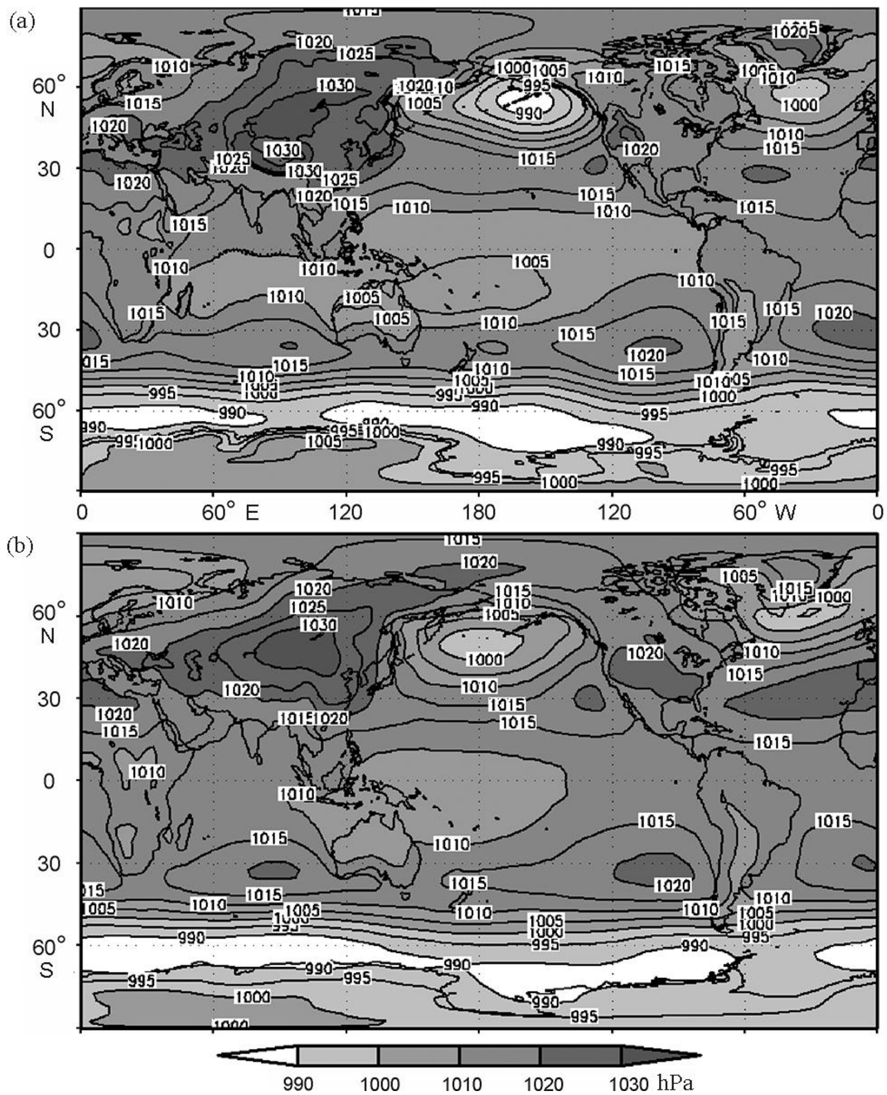

Fig. 3. Averaged for January sea level pressure field (hPa) (a) according to the model and (b) NCEP reanalysis.

An important advantage of the unified model in comparison with the operational version is the more effective use of computing resources. The operational version based on the OpenMP parallelization technology can use up to 16 processor cores (only 8 efficiently) while at this resolution the unified version using a combination of MPI and OpenMP technologies almost linearly scales up to 64 processor cores; there is still noticeable acceleration of the program while using 128 cores in comparison with 64 cores. It is extremely important for the planned use of the model for modeling of climate changes.

Using the unified version of the SL-AV model, we carried out the experiment on atmospheric circulation calculation for 6 years according to the protocol of the AMIP2 international experiment for initial data for January 1, 1979 with known and changing in time fields of sea-surface temperature and sea ice concentration.

The preliminary estimates of the model climatology show that the model reproduces main large-scale features of observed atmospheric dynamics. The average January sea level pressure in the model (presented in Fig. 3) is generally close to the observed one (only the Aleutian pressure minimum is shifted to the north-east in the model and is expressed stronger with respect to observational data). The zonal wind speed in the troposphere (Fig. 4) of the model is also close to observational data. Some overestimate of the eastward wind speed in the tropical stratosphere can be related to the dynamics scheme features at the equator that apparently require adjustment. Emergence of light western winds instead of eastern ones in the tropical lower troposphere can also be related to the same features.

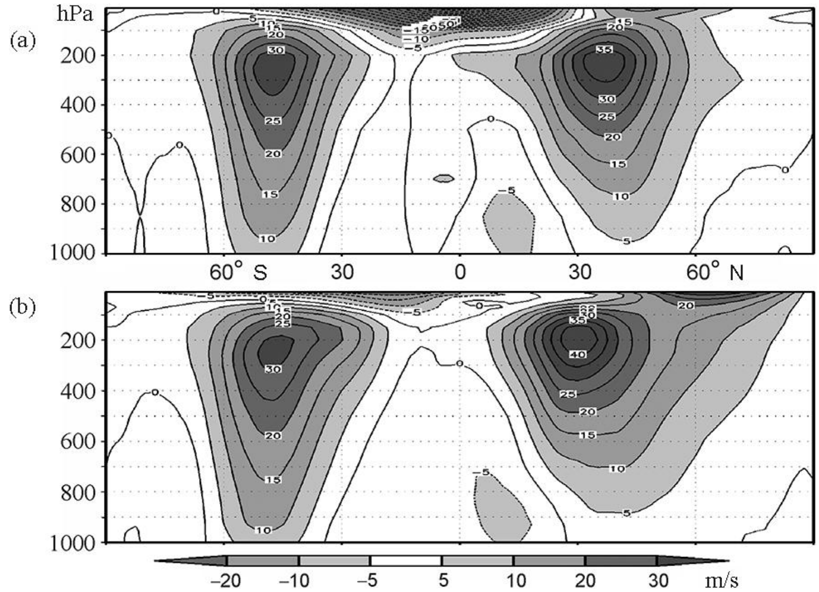

Fig. 4. Average January zonal wind speed field (m/s) averaged along latitude circles, (a) according to the model and (b) ERA reanalysis.

The analysis of the structure for the field of the northward heat flux generated by stationary waves (in winter) allows the following conclusions. In the Northern Hemisphere troposphere the flux reaches 10–15 K m/s that corresponds to the NCEP/NCAR reanalysis data. In the stratosphere, the model underestimates this quantity (35 K m/s in comparison with 60 K m/s in the NCEP/NCAR archive). As the temperature flux is proportional to the vertical flux of the energy of Rossby waves, one can note that in the model stationary waves propagate into the stratosphere less efficiently than in nature. This shortcoming is inherent to many models of the atmosphere general circulation.

In general, the quality of modern climate simulation with the SL-AV model can be considered as good, and apparently the model can be successfully coupled with the ocean model after some tuning. Finally, it is necessary to note that these calculations were carried out with the model version having the aposteriori corrector of the atmosphere mass. Further it is planned to apply the locally conservative version of the model dynamical core [23] for modeling of climate changes.

6. CONCLUSION

The SL-AV global atmosphere model whose development was supported and inspired by G.I. Marchuk for many years, was introduced into the operational practice at the Hydrometeorological Center of Russia. This allowed significant reduction of the gap between Russia and the leading world centers in the quality of medium-range weather forecasts. Thus, the main objective of the model development was achieved.

The work on the standardization of the SL-AV model different versions presented in the article led to the development of the multiscale atmosphere model on this basis. The possibility of applying the SL-AV model to medium-range weather prediction, and, after some revision, for modeling of climate changes is shown.

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