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On the effects of small-scale variability on acoustic propagation in Fram Strait: The tomography forward problem

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Acoustic tomography systems have been deployed in Fram Strait over the past decade to complement existing observing systems there. The observed acoustic arrival patterns are unusual, however, consisting of a single, broad arrival pulse, with no discernible repeating patterns or individual ray arrivals. The nature of these arrivals is caused by vigorous acoustic scattering from the small-scale processes that dominate ocean variability in Fram Strait. Simple models for internal wave and mesoscale variability were constructed and tailored to match the variability observed by moored thermisters in Fram Strait. The internal wave contribution to variability is weak. Acoustic propagation through a simulated ocean consisting of a climatological sound speed plus mesoscale and internal wave scintillations obtains arrival patterns that match the characteristics of those observed, i.e., pulse width and travel time variation. The scintillations cause a proliferation of acoustic ray paths, however, reminiscent of “ray chaos.” This understanding of the acoustic forward problem is requisite to designing an inverse scheme for estimating temperature from the observed travel times.

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I. INTRODUCTION

The application of tomography for observing Fram Strait (Fig. 1) has been examined by theory or simulation in the past (Chiu et al., 1987; Naugolnykh et al., 1998), and the conclusions of those studies suggested a practical utility to such observations. The interest in Fram Strait stems from its role as the only deep-water connection between the North Atlantic and Arctic Oceans and the associated transports of both heat and salt (Fieg et al., 2010; Schauer et al., 2008). The importance of Fram Strait in the world ocean climate system makes it a critical region for a sustained ocean observing system (Sagen et al., 2010; Sandven et al., 2014; Fieg et al., 2010; Mikhailovsky et al., 2015). The turbulent, small-scale processes that characterize these transports make them difficult to observe accurately using available technologies. The Rossby radius of deformation in the region, the theoretical horizontal scale for mesoscale variability, is 4–10 km (Nurser and Bacon, 2014; Fieg et al., 2010). In recent years observations using acoustic tomography have been implemented in Fram Strait (Sagen et al., 2016; Skarsoulis et al., 2010; Mikhailovsky et al., 2015). Not surprisingly, the character of the observed acoustic arrival patterns is significantly different than that described by the previous theoretical studies; the actual ocean is of a different character than that employed by the theoretical studies.

Beginning in 2008–2009 with the “Developing Arctic Modelling and Observing Capabilities for Long-term Studies” (DAMOCLES) program, continuing through 2009–2011 with the “Acoustic Technology for Observing the Interior of the Arctic Ocean” (ACOBAR) deployments, and now sustaining these observations with the present-day “Arctic ocean under melting ice” (UNDER-ICE) program with observations spanning 2013–2016, the Nansen Center in Bergen, Norway has conducted an on-going observation program for Fram Strait focused on exploiting the remote sensing capabilities of acoustics, using both active (acoustic tomography) (Dushaw et al., 2016; Dushaw and Sagen, 2016; Sagen et al., 2016) and passive sources (Geyer et al., 2016; Haugan et al., 2012; Mikhailovsky et al., 2015). Travel-time data for acoustic tomography obtained in the Fram Strait region during the DAMOCLES and ACOBAR deployments has the property that clear, individual ray arrivals are not often observed. Rather, the arrival coda usually consist of a single, O(100 ms) broad, arrival pulse with irregular characteristics, followed by a sparse, random collection of what are evidently bottom-scattered arrivals (Figs. 2 and 3). The tomography system was designed to resolve acoustic signals with O(10 ms) precision (Worcester et al., 2013). Individual ray arrivals are observed in most regions of the world, including the Greenland Sea (Munk et al., 1995). Predictions of the arrivals in Fram Strait using a smoothed ocean atlas show that all the various computed eigenrays arrive at the receiver at about the same time; there is minimal dispersal of the acoustic arrival pattern in this region. The dispersal, or spread, of the predicted arrival
pattern is considerably less than what is observed, however. The nature of the measured arrivals suggests vigorous acoustic scattering is occurring, as is commonly observed near the finale of mid-latitude arrival patterns (e.g., Colosi et al., 2005). Understanding the nature of the observed arrival patterns is an essential part of the process of developing procedures to estimate temperature variations by inversion of the acoustic data (Dushaw et al., 2016; Dushaw and Sagen, 2016; Sagen et al., 2016); this is the forward problem of ocean acoustic tomography (Munk et al., 1995). The conjecture of the analysis described here is that if scattering of acoustic energy by small-scale variability is an important aspect of the observed arrival patterns, then properly addressing this scattering may lead to more accurate formulations of the inverse problem, hence more accurate estimates of ocean temperature.

The aim of this analysis was therefore to examine the issue of the scattering of acoustic rays by the small-scale variability that characterizes the oceanographic conditions of the Fram Strait region to see if the observed arrivals could be more accurately modeled. Further, clarifying the nature of the acoustic characteristics may lead to better signal processing that might result in clearer arrivals, or perhaps acoustic scattering inherently precludes any improvement in the arrival patterns through signal processing.

For this study, the 130-km acoustic path of DAMOCLES program was used. The 2009 World Ocean Atlas (WOA09) (Antonov et al., 2010; Locarnini et al., 2010) was used to obtain a climatological-mean sound speed section for this path. The temperature variability was determined using observations by the Alfred Wegener Institute (AWI) and Norwegian Polar Institute (NPI) Moored Array (Schauder et al., 2008; von Appen et al., 2015) (Moored Array, hereinafter). This array has been deployed across Fram Strait since 1997. The observed variations of ocean sound speed were separated into short and long time scales. While the initial hypothesis was that the short time scale variability was caused by internal waves, this hypothesis proved to be mostly incorrect. The model for internal waves employed here was developed by Colosi and Brown (1998). The stratification in Fram Strait is such that internal wave contributions to sound speed scintillations are quite weak. Variability at both short and long time scales likely resulted from smaller and larger mesoscale variability, respectively. Models for the mesoscale variations at smaller and larger scales were developed, employing approaches similar to those for objective maps (Cornuelle et al., 1989; Cornuelle et al., 1993; Morawitz et al., 1996). The effects of the smaller- and larger-scale variations on the acoustic propagation were assessed separately.
and together. The effects of these variations were found to give a single-lump arrival pattern that is similar to those observed.

In Secs. II and III, the DAMOCLES tomography and the Moored Array programs are, respectively, described. The internal wave and smaller mesoscale models that were employed for simulating the ocean variability at the small scale are described in Sec. IV, and the larger mesoscale model is described in Sec. V. The results of acoustic computations using ray tracing that simulate the acoustic propagation through the ocean are discussed in Sec. VI. The computations equivalent to rays using the parabolic equation are given in Sec. VII. A brief summary and discussion are given in Sec. VIII.

II. DAMOCLES TOMOGRAPHY

The DAMOCLES acoustic source, moored at 378 m depth, began transmissions in September 2008 that continued through July 2009 (Fig. 2). The signals were received by the four receiver hydrophones moored at 685, 781, 877, and 973 m depths at 130.012 km range. The source mooring was located at 78.5105°N, 8.2516°E with a depth of about 1570 m, and the receiver mooring was located at 78.4259°N, 2.4412°E with a depth of about 2390 m. After recovery, the signals were processed by pulse compression techniques to form the equivalent impulse response signal. The travel times were corrected for clock drift and the motion of the source and receiver moorings. After the processing and corrections, the recorded travel time of any given pulse could be determined to about 10 ms precision. The acoustic time series recorded by the four hydrophones are all similar. Gaps in the record were caused by a programming error in the receiver instrumentation that led to an occasional saturation of the acoustic signal.

While acoustic tomography normally relies on the resolution of individual ray arrivals, such arrivals were not evident in the data obtained (Figs. 3 and 4). The arrivals of acoustic rays are typically evident as a sequential pattern of clear, narrow peaks of acoustic energy, repeated over the entire record. Once a computed arrival pattern is correlated with...
with a measured arrival pattern, the ray arrivals are most often identifiable; this identification determines the spatial sampling associated with each ray arrival. The arrival patterns observed in Fram Strait do not often exhibit regular, repeatable ray arrivals. Figure 4 gives examples of three arrival patterns obtained at three different times. The only repeatable feature in these patterns is the overall lump of acoustic energy, with the times of largest amplitudes spanning about 100 ms. Ray arrivals observed in mid-latitude experiments are typically single peaks of O(10 ms) duration (e.g., Worcester et al., 1999). Representations of tomo- 
graphic acoustic time series are often shown as “dot plots,” which show just the times and amplitudes of the local peaks of acoustic energy. Selected peaks associated with the arrival patterns of Fig. 4 are indicated, and the dot plot equivalent of Fig. 3 is given in Fig. 5. In this case, the selected peaks give only an approximate indication of the complete arrival pattern; the dot plot omits considerable information about the nature of that pattern. The dot plot approach is most useful in cases in which clear ray arrivals are evident.

An arrival pattern equivalent to that observed was computed using a sound speed section computed from WOA09 (Antonov et al., 2010; Del Grosso, 1974; Locarnini et al., 2010). An alternate climatology was the Polar science center Hydrographic Climatology (PHC; version 3.0) (Steelle et al., 2001). This climatology does not have the awkward singularity at the North Pole that the WOA09 has, but it is not as smoothed in the vertical as the WOA09. The lack of vertical smoothing in PHC introduces artificial discontinuous sound speed gradients that make it unsuitable for acoustic calculations (e.g., Dushaw et al., 2013). Bathymetry along the acoustic path was derived from the International Bathymetric Chart of the Arctic Ocean data base, with 2 min resolution (Jakobsson et al., 2008). Acoustic propagation in Fram Strait is bottom limited to around 2000 m, and on this section the depth varied from 1500 to 3000 m (Fig. 6). The calculation of rays from the DAMOCLES source to receiver produced just a pair of surface-reflecting, refracting (RSR) rays, and a small set of surface-reflecting, bottom-reflecting rays. The predicted “timefront,” or the ray arrival pattern shown as a function of depth and travel time, shows the two RSR ray arrivals at about the 89.1 s travel time, with a few bottom reflecting rays arriving later (Fig. 7). The conjecture of this paper is that the main group of observed arrivals are associated with these two RSR arrivals, but that the individual identity of these rays is lost by vigorous acoustic scattering.

The time series of recorded arrivals varies in travel time by up to 200 ms. Dominant oceanographic variations are due to the mesoscale variations at weekly to monthly time scales (von Appen et al., 2015) and the seasonal cycle with warming (decreasing travel times) in June and July (with maximum temperatures in September–October; Beszczynska-Möller et al., 2012). The magnitude of the seasonal cycle is enormous when compared to comparable variability observed at midlatitudes (Dushaw et al., 1993). The decrease in travel
time observed during summer on the 130-km DAMOCLES acoustic path is about 200 ms. Roughly the same decrease was observed in the central North Pacific by tomography in 1987 on acoustic paths an order of magnitude longer (Dushaw et al., 1993). In the central North Pacific, the warming was caused by the formation of the summer mixed layer, a 50–100 m thick near-surface layer that formed between May and September 1987 from solar radiation. Given the short DAMOCLES acoustic path, the observed variations in heat content are of an order of magnitude larger than the Pacific summer mixed layer. In Fram Strait, the observed warming results from the combined effects of the advection of warm water by the West Spitsbergen Current (WSC) and solar insolation in the near-surface layer.

III. THE MOORED ARRAY TIME SERIES

The next steps in this analysis were to construct models for internal wave variability and the extraordinary mesoscale variability that dominate the region. The Moored Array data provided benchmark measurements for the magnitude of the fluctuations in sound speed caused by internal waves and the mesoscale. The internal wave and mesoscale models discussed next were adjusted so that their variations roughly matched the observations. It is, of course, impossible for these simplified models to be a precise description of the Fram Strait oceanography. We seek only to model the variations approximately, so that the scintillations they impart to the acoustic predictions are at least about right. The basic nature of acoustic predictions is likely relatively insensitive to the details, or absolute correctness, of the modeled variations.

Time series of temperature and ocean currents have been measured by the Moored Array since 1997. The Moored Array consisted of about 16 moorings deployed across Fram Strait along 78.83°N from the East Greenland shelf to the shelf west of Svalbard. The spacing between the moorings is about 7 km in the upper slope, about 21 km on the lower slope, and 25 km in the deep section (Fig. 1) (Beszczynska-Möller et al., 2012; Schauer et al., 2008; von Appen et al., 2015). The data include time series of temperature sampled at 15-min (SBE sensors) or 2-h (Aanderaa current meters with temperature sensors) intervals, obtained at four to six depths on each mooring. The sampling in the vertical on these moorings is sparse. Time series on several of the moorings from 2008 to 2009 were examined and filtered to separate high- and low-frequencies using a 2-day running mean. This filtering was used to try to separate internal waves and internal tides with periods of minutes to O(1 day) from the mesoscale and other variability with longer periods. As will be described in Sec. IV, the high-frequency variability cannot be accounted for by internal waves, however. Figures 8(a) and 8(b) show the high- and low-frequency time series obtained on mooring F6 at 78.8°N, 5.0°E located at the edge of the West Spitsbergen Current offshore branch (Beszczynska-Möller et al., 2012) where mesoscale activity is a dominating oceanographic feature. This mooring is influenced from the north by a quasi-permanent eddy around the Molloy Deep and to the south by topographically steered currents at the Knipovich Ridge. The instruments on mooring F6 were at 70, 252, 1504, and 2635 m depths. Time series from other moorings in the WSC offshore branch are similar while in the WSC core high-frequency variability is less pronounced.

In the upper ocean (ca. 70 and 250 m) the variations in sound speed were about 2 m s⁻¹ root-mean-square (RMS) for both high- and low-frequencies. In this region, a 1 °C temperature variation corresponds to 4.5 m s⁻¹ sound speed variation (Dushaw et al., 2016). Taylor’s Hypothesis was tacitly assumed, that is, the RMS of the time variations was assumed to be equivalent to the RMS of the spatial variations.

The variability declined considerably below 750 m depth, where high-frequency sound speed RMS was about 1 m s⁻¹. Most of the variability is associated with the West Spitsbergen Current, which does not extend below about 1000 m. In the deep regions of Fram Strait observed peak-to-peak temperature variations were about 0.1 °C (with the climatological trend decreasing this variability) (von Appen et al., 2015), corresponding to about 0.5 m s⁻¹ in sound speed.

IV. INTERNAL WAVE AND SMALL MESOSCALE MODELS

Models designed for the purposes of computing the effects of internal waves on acoustic propagation have been around for many years (e.g., Flatté et al., 1979). While never a precise description of internal waves actually in the ocean, such models have been established as tools for addressing the issues of internal wave effects on long-range acoustic propagation. Van Uffelen et al. (2010) used the model of Colosi and Brown (1998) to show that the acoustic scattering by internal wave scintillations was the mechanism causing extension of the cusps of acoustic time fronts deep into the shadow zone. These models approximate the Garrett-Munk spectrum of internal waves, although they do not distinguish between fluctuations caused by internal waves and those caused by spice.
A. The Colosi-Brown internal wave model

The Colosi-Brown model was used to construct a model for internal waves within Fram Strait using the climatological profiles for buoyancy, temperature and sound speed, and a latitude of 77°C14 N. Attempts to adjust the parameters of the model so that the resulting internal wave variations had the approximate magnitudes measured by the Moored Array proved futile, however. The stratification within Fram Strait is such that entirely unphysical vertical displacements are required to give temperature variations that match the observations. Figure 9 shows the climatological temperature, buoyancy, and the resulting RMS sound speed fluctuations as a function of depth obtained using realistic parameters for the internal-wave model. The fluctuations modeled this way are much weaker than the observations. The parameters used for the model were a thermocline depth $B = 1000\,\text{m}$, spectrum indices $j_{\text{max}} = 100$ and $j_* = 3$, and $\zeta_0^2 = 32$, appropriate for a normal Garrett-Munk internal wave spectrum. $N_0$, the reference buoyancy, was computed as the depth average buoyancy with value $0.0026\,\text{rad}\,\text{s}^{-1}$. The model has a thin layer of high variability corresponding to the base of a near-surface mixed layer, and weak variability between 100 and

FIG. 8. (a) The time series of temperature from the F6 mooring of the Moored Array. The time series for instruments at 70, 252, 1504, and 2635 m were separated into: (a) the high-frequency time series ($>0.5\,\text{cpd}$), and (b) the low-frequency time series. Temperatures to vary by up to $\pm 2\,\text{°C}$ in the upper ocean. Below about 1000 m, temperature variations are greatly reduced; temperatures at 1504 m vary by only $\pm 50\,\text{m}\,\text{°C}$, corresponding to $\pm 0.23\,\text{m}\,\text{s}^{-1}$ sound speed.

![Graphs showing temperature profiles and frequency](image1)

![Graph showing RMS sound speed fluctuations](image2)

FIG. 9. (left) Temperature profile and (middle) buoyancy frequency for the DAMOcles acoustic path derived from WOA09. (right) RMS internal-wave variations of sound speed derived from the Colosi-Brown internal wave model. The internal wave model employed a normal Garrett-Munk internal wave strength.
Substantive studies of internal waves within Fram Strait are lacking, with perhaps the near-surface analysis of Sandven and Johannessen in 1987 (Sandven and Johannessen, 1987) being what is available. See also Sarkar et al. (2015). The conclusion reached here regarding the weak contributions of internal waves is consistent with that reached by Dyer et al. (1987). The properties for internal waves of the region may not be optimally modeled by this mid-latitude formalism. A more careful assessment of the model within Fram Strait, and perhaps an extension of the model to account for spice and inertial effects, is an area for future research.

B. The small mesoscale model

Given the failure of the internal wave model to account for the high-frequency variations in sound speed, an alternate interpretation for these variations was required. Such variations are obviously not “dynamical,” that is, induced by vertical displacements, so we surmise they are advective in origin. Typical frequencies of the variations apparent in the Moored Array observations are 1–2 cpd. If a nominal barotropic current strength is about 10 cm s$^{-1}$ (Sarkar et al., 2015), these temporal variations correspond to spatial variations of about 4–8 km, consistent with estimates for the expected Rossby radius. We therefore make the conjecture that the observed high-frequency variations were caused by the advection of small mesoscale features across the Moored Array mooring.

The ocean models used to construct realizations for both the small and large mesoscale environment were similar to that used for ocean estimation by inverse of the acoustic travel times (Sagen et al., 2016). Separate models were constructed for the small and large mesoscale environments. The ocean variability was assumed to be separable into vertical and horizontal functions; the approach is objective mapping (e.g., Cornuelle et al., 1989; Cornuelle et al., 1993; Morawitz et al., 1996). The model was therefore composed of vertical modes and horizontal sinusoids,

$$F(r, z) = \sum_{i=1}^{N_v} V_i(z)[A_i^v + \sum_{j=1}^{N_h} (A_j^h \cos(K_j r) + A_j^h \sin(K_j r))]
= \sum_{n=1}^{N} A_n P_n(r, z).$$

The vertical modes, $V_i(z)$, are empirical orthogonal functions (EOFs) of an ad hoc covariance matrix. For the small mesoscale environment, this matrix is composed of a full water column form

$$C(z_1, z_2) = 4e^{-sz_1/550} e^{-dz^2/10sz},$$

where $sz = z_1 + z_2$ and $dz = z_1 - z_2$. This covariance was tuned to approximate the small-scale variability and covariance characteristics of Fram Strait. The vertical e-folding scale of variance, obtained by setting $z_1 = z_2$, is 275 m. Fifty vertical EOFs computed from a Singular Value Decomposition (SVD) of the covariance were used.

As noted above, the Rossby radius of deformation is about 4–10 km in this region, and the horizontal scales of the

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**FIG. 10.** (Color online) (top) Sound speed along the DAMOCLES section derived from WOA09. (upper middle) A realization of the model for large mesoscale variations. (lower middle) A realization of the model for small mesoscale variations. (bottom) A realization of the model for internal wave variations. The internal wave model has large variations at the base of a near-surface mixed layer at around 50 m. These expected internal wave variations are an order of magnitude weaker than the observed variations, however.

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200 m depths resulting from a weak temperature gradients at this depth. Such characteristics are, of course, dependent on the details of the climatological model, hence the are likely artificial. To obtain fluctuations comparable to the observations, a $\zeta_0^2$ value of about 6300 was required, 2 orders of magnitude larger than is physically possible. The climatological buoyancy was compared to buoyancy profiles computed from 13 CTD casts obtained along the DAMOCLES path in August 2009 and found to be only incidentally different.

Figure 10 shows the climatological sound speed section along the DAMOCLES acoustic path, indicating the core of the West Spitsbergen Current near the acoustic source. The bottom panel of this figure gives a realization for the surpris-

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simulated small mesoscale were assumed to be this size. These scales are not precise. Available hydrographic sections have not been obtained at sufficiently close intervals to accurately determine the statistics of the variability, certainly not at these small scales. Typical hydrographic profile spacing has been 18–20 km, comparable to the spacings of the Moored Array.

The form for the wavenumber spectrum of horizontal variability employed here was

\[ S_{ij} = \frac{\omega_i}{K_i^2 + K_0^2} = \langle A_{ij}^2 \rangle = W_n. \]  

(3)

This form is derived from the assumption that the spatial covariance has an exponential falloff with range. The indexes \( i \) and \( j \) refer to vertical and horizontal function indexes. To account for the quite small scales, 1500 wavenumbers were employed, with \( K_j = 2\pi j/L \) and \( L \) equal to 1.5 times the section range. The \( K_0 \) determine the horizontal scales associated with the \( n \)th vertical functions, with \( K_0 = K_0^L + 20 \). The form for the \( K_0 \) was determined in an ad hoc fashion to obtain a simulated variability that roughly approximates the O(4–8 km) expected length scales (Fieg et al., 2010). Higher-order vertical EOFs have shorter scales. The \( W_n \) are the variances associated with the \( n \)th two-dimensional (2-D) functions. The overall level of the wavenumber spectrum associated with each vertical function, \( w_i \), is set by the EOF spectrum.

Using the variances of the amplitudes determined by the EOF and wavenumber spectra, statistically consistent snapshots of the small mesoscale ocean were readily derived. Mesoscale snapshots were derived using amplitudes for the 2-D functions given by

\[ A_n = \sqrt{W_n} p_n, \]  

(4)

where \( p_n \) is a random number with Gaussian distribution and unit standard deviation. The computed amplitudes, \( A_n \), were then applied to each 2-D function in the equation for \( F_L(r, z) \) above to obtain simulated snapshots of small-scale ocean states.

V. THE LARGE MESOSCALE MODEL

The ocean model used to construct realizations of the large mesoscale environment was similar to that used for the small mesoscale environment. The DAMOCLES acoustic path is in an ice-free region of Fram Strait, hence eddies associated with the ice edge (Johannessen et al., 1987; Mellberg et al., 1991; Sandven et al., 1991) were not a factor. As above, the model was composed of vertical modes and horizontal sinusoids,

\[ F_L(r, z) = \sum_{i=1}^{N_L} V_i(z) [A_0^i]^2 \]

\[ + \sum_{j=1}^{N_L} (A_j^i \cos(K_j r) + A_j^i \sin(K_j r))] \]

\[ = \sum_{n=1}^{N} A_n P_n(r, z). \]  

(5)

The vertical modes, \( V_i(z) \), are also empirical orthogonal functions (EOFs) of an ad hoc covariance matrix. This matrix is composed of a full water column form

\[ C(z_1, z_2) = 20e^{-z_1/750} e^{-d^2/1000}, \]  

(6)

where \( dz = z_1 + z_2 \) and \( dz = z_1 - z_2 \), is 375 m. Values for the near-surface of this covariance were then scaled using a function \( \sqrt{1 + 2(1 - \tanh(z/50))} \) to allow for variations of a near-surface mixed layer. This function has a value of about 1.7 at the surface, and it falls rapidly in value to about 1.0 below 150 m. Since it is applied to both dimensions of the covariance, the assumed variance of sound speed at the surface is increased from 20 to about \( (1.7)^2 20 = 58 \text{ m}^2 \text{s}^{-2} \) (or 7.6 m s\(^{-1} \) RMS). Below about 150 m, the covariance is given by the form above.

The spectrum of the EOFs, \( \{w_i\} \), gives the variance associated with each EOF, and determines the modeled oceanic variance. This spectrum falls off rapidly enough that just the first five EOFs were deemed necessary to capture most of the larger-scale variability, that is, the variance associated with the sixth EOF was about 20 times less than the first EOF.

FIG. 11. (left) Simulated variations of the small mesoscale as a function of range at depths corresponding to the observations shown in Fig. 8(a). The amplitudes of these variations are designed to be similar to the variations in time observed at the F6 mooring. (right) The RMS sound speed as a function of depth obtained from the small mesoscale simulation.
While, as noted in the introduction, the Rossby radius of deformation is about 4–10 km in this region, the horizontal scales of the actual variability of the region are not well known. For our purposes here we assumed a horizontal scale of $O(10 \text{ km})$ for the large mesoscale.

As above, the form for the wavenumber spectrum of horizontal variability was

$$S_{ij} = \frac{W_i}{K_j^2 + K_0^2} = \langle A_n^2 \rangle = W_n.$$  \hfill (7)

Fifty wavenumbers were employed, with $K_j = 2\pi j/L$ and $L$ equal to 1.5 times the section range. With $K_0 = 2\pi/X_i$, the $X_i$ determine the horizontal scales of the $i$th vertical functions. The values for the $X_i$ were determined in an ad hoc fashion to be 200, 100, 50, 30, and 10 km; note that these $X_i$ are wavelengths, which are not equivalent to spatial scale. These lengths were chosen to give simulated variability that roughly approximates the $O(10 \text{ km})$ length scales that characterize the large-scale variability of Fram Strait (Fieg et al., 2010).

Using the variances of the amplitudes determined by the EOF and wavenumber spectra, statistically consistent snapshots of the large mesoscale ocean were readily derived, as described above for the small mesoscale environment.

The upper middle panel of Fig. 10 gives an example of the large mesoscale environment modeled this way. The sound speed variations ($\pm 9 \text{ m s}^{-1}$) are roughly consistent with the low-frequency variations observed by the Moored Array. The summation of the climatological mean sound speed section, the large and small mesoscale models, and the internal wave model give an example of what the Fram Strait acoustic environment might look like (Fig. 12), under the assumptions employed by this study. Any number of such realizations were readily computed.

VI. RAY TRACING PREDICTIONS

Using any given sound speed section, it is a simple matter to trace rays through it to obtain predictions for the acoustic arrival patterns for any source and receiver combination. To assess the relative contributions of the climatological state, the large mesoscale and small scale on acoustic propagation, time fronts were computed using the sound speed sections of Figs. 10 and 12 (Fig. 13). The small
mesoscale and internal wave realizations were combined and will be referred to as small-scale realizations. Given the short scales of the small-scale models, the ray calculations were made as accurate as possible by using appropriate ray-tracing parameters. Both the step size of the Runge-Kutta integrations and the error limits of those integrations were greatly reduced (Dushaw and Colosi, 1998). Bottom-reflecting rays were omitted from the calculations.

The ray arrival pattern for the smoothed ocean atlas is well-behaved for the most part [Fig. 13(a)], although the branches of the time front are reversed compared to the usual midlatitude pattern, with the deeper-turning rays arriving later. This optimistic pattern is greatly influenced by the large mesoscale variations [Fig. 13(b)]. It seems apparent that, given this model for the mesoscale, the precise structure of the arrival pattern depends on the precise state of the mesoscale; the arrival pattern is different for each realization of the mesoscale environment. The mesoscale state also affects the overall travel time of the arrival pattern. The usual mid-latitude mesoscale environment does not affect acoustic arrival patterns this way. Mesoscale variations in midlatitudes hardly affect the form of the arrival pattern, but the overall travel time changes in response to the warming or cooling from mesoscale features.

The sound speed fluctuations from the small-scale model significantly affected the acoustic propagation [Fig. 13(c)]. The ray calculations indicated fairly vigorous scattering of the ray arrivals, causing the nearly total loss of the branches of the time front, and a broadening of the acoustic arrival pattern. This broadened, scattered pattern appears to correspond to the 100-ms wide arrival pulse measured during DAMOCLES (Fig. 2). The arrival pattern obtained from the combined large mesoscale and small-scale models [Fig. 13(d)] bore little resemblance to the pattern derived from the climatological ocean.

To test how realistic the simulated environments were, ray predictions were computed in ten sound speed sections with combined large mesoscale and small-scale realizations. These realizations were independent, and no time dependence was implemented. Travel times between the DAMOCLES source at 378-m depth and receiver at 973-m depth were obtained for these ten snapshots (Fig. 14). Both the variations in the travel times of the cluster of arrivals and the variations in the width of the clusters are reasonably consistent with the measurements shown in Figs. 2, 3, and 5. The conclusions from this comparison, together with the comparisons to the Moored Array data, is that the large mesoscale and small-scale models employed here approximate the variability of the actual ocean, at least insofar as the limited available data can determine, and the small scales of the variability within Fram Strait scattered the acoustic signals to form the single, broad arrival patterns that were observed.

The nature of the arrivals in the time fronts suggests that the small-scale variations in the ocean have induced a proliferation of rays. Indeed, the effects seen here may be examples of “ray chaos” (Smith et al., 1992a, 1992b; Simmen et al., 1997). To determine the effects of the sound speed fluctuations on the ray paths, eigenrays were computed between the source at 378-m depth and the receiver at 973-m depth. Figure 15 (top) shows the two eigenrays obtained using the climatological sound speed section, while Fig. 15 (bottom) shows the many eigenrays obtained from the climatological section including mesoscale and internal wave variations. This result suggests that ray theory has broken down in this particular environment, or perhaps more technically, the Fresnel zones have become so large that their association with particular rays is no longer meaningful. Inversion of these data for average ocean temperature might be best achieved through the use of “travel-time sensitivity kernels” (Skarsoulis et al., 2009), which rigorously account for the sampling of the acoustic field associated with a particular arrival pulse. One interpretation is that this result suggests the recorded arrival pulses measure an almost ideal average of temperature along the section.

VII. PARABOLIC EQUATION PREDICTIONS

The effects of internal waves on long-range acoustic transmissions are often computed using the parabolic equation technique (PE) (Jensen et al., 1994). This approach...
gives a complete solution for the acoustic field. Given the nature of the sound speed perturbations employed here, there was the possibility that the ray approach may be inadequate; any limitations in the ray approximation are overcome with the parabolic equation (Simmen et al., 1997).

The arrivals computed using the parabolic equation for the specific example of Fig. 12 are given in Fig. 16. The effects of the sound speed scintillations on the arrival pattern are similar to those obtained by geometric rays. The predicted arrival, a single fairly broad pulse arrival of perhaps 100 ms duration, is consistent with the measured arrivals of Fig. 2. Both the ray and PE predictions suggest an individual, resolved ray arrival might be obtained using a receiver below about 1500 m depth.

VIII. DISCUSSION

The essential result of this study is the illumination of the essential nature of the arrival pulses recorded by the DAMOCLES tomography instruments. The single, broad arrival pulse of perhaps 100 ms width arises from a combination of the basic nature of the sound channel axis in the Fram Strait region and scattering from small-scale oceanic features. Ray tracing and parabolic equation computations gave equivalent results. The sound channel axis is fairly flat in this region resulting in minimal dispersal of the acoustic arrival pattern (Fig. 17). The sound speed gradients of the climatological ocean are overwhelmed by the mesoscale and internal wave variations, resulting in the single, thoroughly scattered arrival pulse. The width of the arrival pulse predicted this way was consistent with that recorded during DAMOCLES. Absent the small-scale variations the precise arrival pattern is determined more by the particular large mesoscale environment than by the climatological ocean; this arrival pattern likely changes completely from one mesoscale realization to another. Further, since the simulated fluctuations in the travel time of the pulse are determined by large mesoscale variations, it appears likely that the similar, observed fluctuations in travel time are also determined by the mesoscale variations. While the details of this analysis may be dependent on the accuracies of the climatological ocean, the large and small mesoscales, or internal waves as modeled for this study, the basic conclusion about the effects of small scale scattering on the nature of the acoustic data acquired in Fram Strait is not dependent on those details.

Supporting the earlier conclusions of Dyer et al. (1987), the internal waves were found to contribute only weakly to the small-scale sound speed scintillations. The stratification within Fram Strait only weakly supports such waves. Rather, the dominant small-scale variations were suggested to be caused by the advection of 4–8 km mesoscale features, consistent with the expected small Rossby Radius for the region. The Colosi-Brown model was used to account for the weak internal waves and these simulations were combined with a simulated small mesoscale environment. This simulated small-scale ocean environment was roughly consistent with available observations.
The small-scale scintillations caused scattering that resulted in a proliferation of eigenrays that essentially fill the water column between the DAMOCLES source and receiver moorings. One interpretation may be that the Fresnel zones associated with the rays became so wide that the notion of individual ray paths no longer has meaning. Inversions of the recorded data to obtain estimates of temperature variations may be more accurately computed using the travel-time sensitivity kernel of Skarsoulis et al. (2009). It seems evident, however, that the recorded pulse travel times represent a measurement of a fairly thorough average of temperature over the upper ocean of the water column and over the acoustic path. In a related paper discussing the inverse problem for these data, Sagen et al. (2016) have applied this interpretation to design an inverse scheme for deriving temperature estimates from these data.

The calculations found a more clearly resolved arrival at about 1650 m depth which suggested that a receiver placed at this depth may record a more well-defined ray arrival. The search for eigenrays that arrive at this depth shows that this more definite arrival is associated with a single deep-turning ray (Fig. 18). The small scale variations scatter this ray considerably, but the essential nature of this ray is preserved. Deployment of a deep hydrophone to record arrivals such as this may be recommended. If such a hydrophone were to record a single, stable ray arrival, such data would represent additional oceanographic information for acoustic tomography.

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FIG. 17. Profiles of sound speed derived from WOA09 at the locations of the DAMOCLES source (solid) and receiver (dashed) moorings. These are the sound speed profiles at 0- and 130-km range of the top panel of Fig. 10. The sound speed gradients of these profiles are comparable to, or less than, the sound speed gradients of the mesoscale and internal wave models. Below about 1000 m, the normal pressure-dependent increase in sound speed dominates.

FIG. 18. Eigenrays computed between the source at 378 m and a deep receiver at 1650 m. (top) Using WOA09, and (bottom) using WOA09 together with mesoscale and internal wave model variations. For this deep receiver, an arrival of a single, deep-turning ray is obtained. Even with the mesoscale and internal wave effects, the essential nature of the ray is preserved. Additional oceanographic information for acoustic tomography.