ENERGY- AND FLUX-BUDGET (EFB) TURBULENCE CLOSURE MODEL FOR STABLY STRATIFIED GEOPHYSICAL FLOWS

S. S. Zilitinkevich [1,2,3,4], **T. Elperin** [5], **N. Kleeorin** [5], **I. Rogachevskii** [5], **I. Esau** [4], and **R. Kouznetsov**[1]

[1] A.M. Obukhov Institute of Atmospheric Physics, Moscow, Russia
[2] Finnish Meteorological Institute, Helsinki, Finland
[3] Division of Atmospheric Sciences, University of Helsinki, Finland
[4] Nansen Environmental & Remote Sensing Centre / Bjerknes Centre for Climate Research, Bergen, Norway
[5] Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer-Sheva, Israel

Detailed discussions of the state of the art in the turbulence closure problem for stably stratified geophysical flows are given in recent literature including our prior papers (Zilitinkevich et al., 2007, 2008, 2009). In brief terms, most of the operationally used closures are based on the budget equation for the turbulent kinetic energy (TKE) per unit mass, E_K, and employ the concept of downgradient transport. The latter states that turbulent fluxes of momentum and scalars are proportional to their mean gradients, so that the closure problem reduces to the determination of the proportionality coefficients in such relations, namely, the eddy viscosity, K_M, eddy conductivity, K_H, or eddy diffusivity, K_D. Kolmogorov (1941) postulated that these coefficients are fully characterised by the turbulent velocity scale identified with the square root of TKE, $u_T = E_K^{1/2}$, and the turbulent dissipation time scale, t_T, or length scale, l, defined by the formulae:

$$\varepsilon_K = \frac{E_K}{C_K t_T} = \frac{E_K^{3/2}}{C_K l}, \tag{1}$$

where ε_K is the TKE dissipation rate, C_K is a dimensionless universal constant (to be determined empirically), and $l \equiv E_K^{1/2} t_T$. Then the straightforward dimensional analysis yielded

$$K_M \sim K_H \sim K_D \sim u_T\, l_T, \tag{2}$$

whereas the omitted proportionality coefficients are dimensionless universal constants.

This approach, although quite successful as applied to neutrally stratified boundary-layer flows, is not immediately applicable to the strongly stable stratification. Indeed, Eq. (2) implies that the turbulent Prandtl number, $Pr_T \equiv K_M / K_H$, can only be a universal constant. On the contrary, numerous experiments, large-eddy simulations (LES) and direct numerical simulations (DNS) demonstrate that Pr_T drastically increases with the strengthening stability (see Figure 2) in obvious contradiction to Eq. (2). This difficulty is traditionally overtaken keeping Eq. (2) but admitting that the turbulent length scales for momentum, l_{TM}, and heat, l_{TH}, are essentially different. In so doing, the Kolmogorov closure, originally formulated for the neutral stratification (where l is proportional to the distance, z, over the surface), to a large extent loses constructiveness and merely expresses unknown stability dependencies of K_M and K_H through unknown dependencies of l_{TM} and l_{TH}. In the EFB closure, we derive the flux-profile relationships from the budget equations for the relevant turbulent fluxes [instead of use of hypothetical Eq. (2)] and, in addition to the TKE budget equation, employ the budget equation for the turbulent potential energy (TPE) per unit mass, E_P.

In the present paper, the first version of the EFB closure (Zilitinkevich et al., 2007) is advanced accounting the difference between the dissipation time scales for TKE and TPE (see Zilitinkevich et al., 2008, 2009), and introducing new, energetically consistent formulation for the turbulent dissipation time and length scales (Zilitinkevich et al., 2010). On this basis we propose a hierarchy of turbulence close models form one-dimensional analytical model of the stationary homogeneous turbulence (shedding new light upon the classical Monin-Obukhov similarity theory) to non-local models of different complexity, based on principally the same physics but employing one, two or more prognostic equations for turbulent statistics – to address different physical and practical problems.

Since Richardson (1920), it was generally believed that in the steady state the velocity shear is incapable of maintaining turbulence (which therefore collapses) when the Richardson number, *Ri*, exceeds some critical value, Ri_c (= 0.25). On the contrary, in the context of turbulence closures, the turbulence cut-off at “supercritical” values of *Ri* (> Ri_c) was considered as artefact and prevented with the aid of “correction coefficients" specifying the ratios $K_M(u_T l_T)^{-1}$ and $K_H(u_T l_T)^{-1}$ as essentially different functions of *Ri* (see Mellor and Yamada, 1974). The EFB closure does not require such corrections and automatically provides maintaining of turbulence by shear at any *Ri* (hence, there is no critical *Ri* in the energetic sense). Moreover, the EFB closure distinguishes between the small-*Ri* and the large-*Ri* turbulence regimes, principally different in nature and separated by the transition zone around *Ri* ~ 0.25 (see Figures 1-4 below) .

Verification of the analytical version of our closure model against observational, LES and DNS data are shown in Figures 1-4, based on data from literature and our own LES of conventionally neutral (CN) and nocturnal stable (NS) boundary layers [LES DATABASE64 described in Esau and Zilitinkevich (2006)].

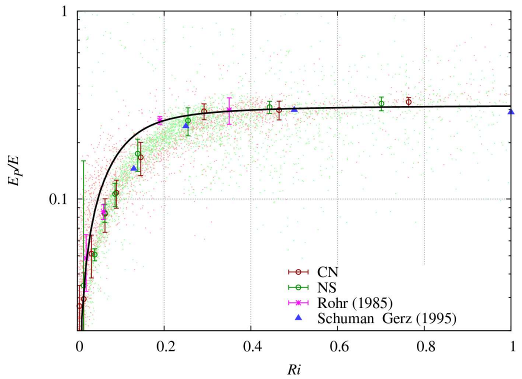

Figure 1: The ratio of potential to total energy versus *Ri*

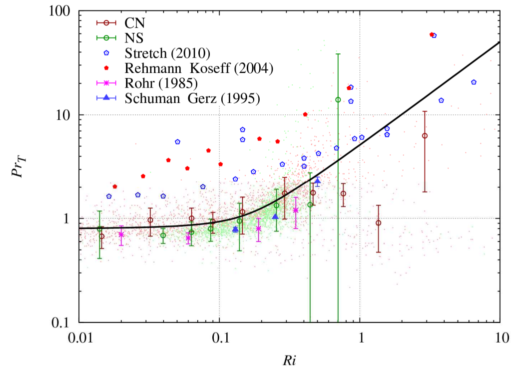

Figure 2: The turbulent Prandtl number versus *Ri*

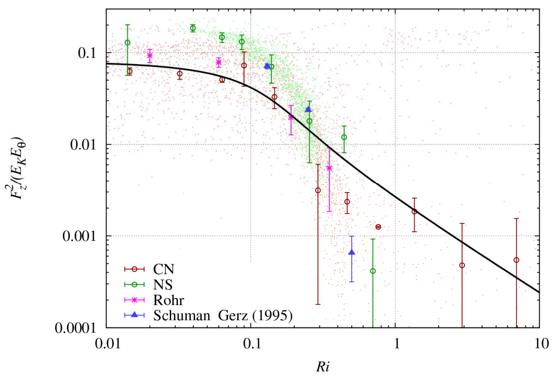

Figure 3: The dimensionless heat flux versus *Ri*

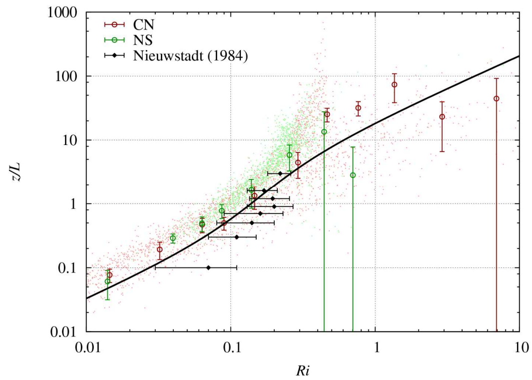

Figure 4: The Monin-Obukhov stability ratio, *z/L*, vs. *Ri*

Ri

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