A RADAR MODULATION TRANSFER FUNCTION FOR OCEANIC INTERNAL WAVES

Knut-Frode Dagestad(1), Vladimir Kudryavtsev(1,2,3), Johnny A. Johannessen(1,4), Morten W. Hansen(1)

(1)Nansen Environmental and Remote Sensing Center, Thormohlens gt 47, 5006 Bergen, Norway, Email:knutfd@nersc.no
(2)Nansen International Environmental and Remote Sensing Center, St. Petersburg, Russia
(3)Marine Hydrophysical Institute, Sebastopol, Ukraine
(4)Geophysical Institute, University of Bergen, Norway

ABSTRACT

Signatures of oceanic internal waves are frequently visible on Synthetic Aperture Radar images. The observed radar backscatter modulations are due to induced surface current variations, but the degree of modulation depend on several factors, such as the phase speed of the internal waves, the ambient wind and viewing geometry. Dimensional analysis is here used to derive simplified expressions relating the observed radar backscatter modulation to relevant geophysical parameters. The derived expressions agree well with simulations with a sophisticated radar imaging model, which is used to determine the free parameters.

1. INTRODUCTION

Internal solitary waves (ISW) propagating on the ocean pycnocline induce surface currents which can reach 1 m/s or more. These currents modify the small scale surface roughness, and for this reason the ISW can be observed in Synthetic Aperture Radar (SAR) images [1]. One example from the South China Sea is shown in Figure 1. The degree of the roughness modulation, as expressed by the amplitude of the normalised radar cross section (NRCS) across the ISW signature, is related to the strength of the ISW, i.e. its amplitude, length scale and induced surface current. However, this relationship is also strongly dependent on the SAR imaging geometry and radar wavelength, as well as surface wind speed and direction.

Our aim is to find a relationship between the surface current induced by oceanic internal waves (solitons) and the modulation of the NRCS which can be measured with SAR. Such a relationship will depend on several factors, and we will here use dimensional analysis to establish a simplified expression.

2. SIMULATED RADAR SIGNATURES OF INTERNAL WAVES

A sophisticated radar imaging model (hereinafter referred to as “RIM”) described in [2], [3], [4] and [5] will be used to simulate the NRCS modulations induced by internal solitary waves. This model consists of a hydrodynamic module and a radar module. The hydrodynamic module calculates a wave spectrum based on input of geophysical parameters such as wind speed and direction; stability of the atmospheric boundary layer; ocean surface current and surface films. Based on this wave spectrum, the radar module calculates the radar return (for both VV and HH polarisations) due to quasispecular reflection, Bragg scattering and breaking waves.

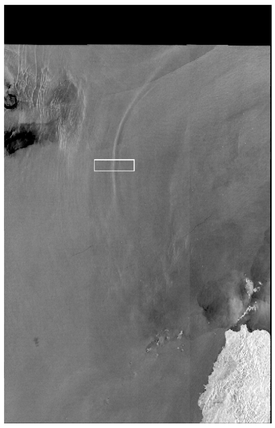

Figure 1: Signature of an internal solitary wave (soliton) on an Envisat ASAR scene from the South China Sea acquired 18 May 2006. Transects within the white box are shown in Figure 2.

The surface current induced by a soliton as derived from classical two layer Korteweg-deVries (KdV) theory [6] is given by:

$$u(x) = u_{\max} \operatorname{sech}\left(\frac{2\pi x}{L}\right), \quad (1)$$

where u_{max} is the maximum induced surface current, x is horizontal distance, and L is a parameter characterising the width of the soliton. The soliton is here assumed to be of depression type, which is by far the most common. For the case of Figure 1, polarisation is vertical, incidence angle is 30 degrees, and L is estimated to 2000 m. According to the NCEP GFS forecast model, the wind direction was inclined by 70 degrees from the IW propagation direction, which was directly towards the SAR sensor. Based on this and surface current given by Eq. 1, the NRCS is simulated for various values of u_{max} and wind speeds, as shown in Figure 2.

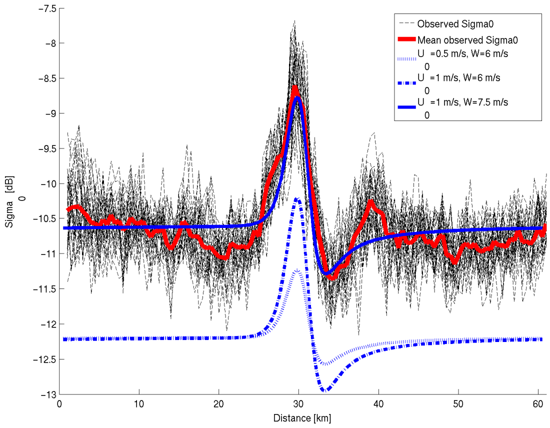

Figure 2: Black lines are 11 adjacent transects of NRCS across the ISW profile of Figure 1, and the red line is their mean. The blue lines are simulated NRCS with RIM for wind speed and maximum ISW induced surface current as given in legend.

It is seen that very good agreement with the observed NRCS is obtained for u_{max} of 1 m/s and a wind speed of 7.5 m/s. The actual surface current induced by the ISW is unknown, but an actual wind speed of 7 m/s given by the NCEP model indicates that the simulated NRCS is realistic.

3. A RADAR MODULATION TRANSFER FUNCTION

We will in this section use dimensional analysis to find a simplified relation between the observed NRCS and the relevant geophysical parameters. Thus we will first construct some nondimensional parameters. Two length scales relevant for internal solitary waves are the characteristic width (L) and the relaxation length of the surface waves which are modulated by the induced current gradients. The distance (D) between the minimum and maximum NRCS values can be measured directly from the SAR image. Assuming that the maximum and minimum NRCS correspond to largest current gradients (negative and positive), it can easily be shown by differentiation of Eq. 1 that D is proportional to L by a factor of 1.32. Hence L can be estimated directly from a SAR image.
The relaxation length is however not a directly visible parameter, and must be parameterized. The NRCS is highly sensitive to ocean waves of the same scale as the radar wavelength, which is typically 2-20 cm for most SAR instruments. Such short waves propagate normally too short distances to be directly modulated by current gradients from internal waves, see e.g. [5]. However, longer waves on meter scale are modulated, leading to enhanced wave breaking, contributing directly to radar scattering, and also indirectly by generation of shorter Bragg waves. Hence we will find here the relaxation length of the shortest waves which are modulated by the current. Consistent with [3], [4] and [5], the relaxation length will be defined by:

$$\lambda = c_g T \,, \qquad (2)$$

where c_g is the group velocity of the modulated surface waves, and T is the relaxation time defined by:

$$T \equiv \frac{1}{\omega\beta} \,, \qquad (3)$$

where ω is the angular frequency of the surface waves, and β is a wind growth rate parameter traditionally parameterized by:

$$\beta = c_\beta \left(\frac{u_*}{c}\right)^2 . \qquad (4)$$

Here c_β is a dimensionless parameter which can be parameterized or determined empirically, c is the phase speed of the surface waves, and u_* is the wind friction velocity. Inserting (3) and (4) into (2) we get for the relaxation length:

$$\lambda = \frac{c^3}{2c_\beta {u_*}^2 \omega} = \frac{g}{2c_\beta {u_*}^2 k^2} \,, \qquad (5)$$

where k is the wavenumber of the surface waves, and we have used the relation $c = \sqrt{g/k} = 2c_g$ valid for deep water.
A dimensionless parameter may then be formed by the ratio of these two length scales:

$$\frac{\lambda}{L} = \frac{g}{2c_\beta {u_*}^2 k^2 L} . \qquad (6)$$

In practical situations, the friction velocity will not be known accurately, and hence we will use the wind speed W at 10 m height, as a substitute for u_*. Then, by removing the dimensionless numbers, we form the following dimensionless parameter:

$$\pi_1 \equiv \frac{g}{k^2 W^2 L} . \qquad (7)$$

For the propagating internal wave soliton, two velocity scales are relevant; the phase speed c_{iw}, and the induced surface current velocity. This induced current varies across the IW profile according to Eq. 1, and has a maximum (u_{max}) above the IW trough. Hence we can form a second nondimensional parameter by:

$$\pi_2 \equiv \frac{u_{\max}}{c_{iw}}. \qquad (8)$$

The modulation of the radar backscatter due to the IW will here be quantified by

$$\Delta\sigma \equiv 10\log_{10}\frac{\sigma_{\max}}{\sigma_{\min}}. \qquad (9)$$

Finally, we assume the following simple relation between the roughness modulation and the two nondimensional parameters:

$$\Delta\sigma = \alpha\pi_1\pi_2, \qquad (10)$$

where α is a parameter which can be determined from measurements or numerical simulations. We did not above consider the strong dependence of the wind growth rate parameter on wind direction relative to the ISW propagation direction, and hence this dependence is lumped into the α parameter. The nondimensional parameters could here for generality be raised to any rational power, but as will be seen, taking these as 1 is adequate. Eq. 10 may alternatively be expressed in term of a transfer function:

$$T \equiv \frac{\Delta\sigma}{\pi_2} = \alpha\pi_1. \qquad (11)$$

For a fixed radar wavenumber and incidence angle, we hence expect the relationship

$$\frac{\Delta\sigma}{u_{\max}/c_{iw}} = \frac{const.}{LW^2}. \qquad (12)$$

To test the validity of Eq. 12, we simulate $\Delta\sigma$ with the RIM model for all possible combinations of the following parameter ranges:

- $u_{\max}$ = 0.2, 0.5, 1 m/s
- L = 200, 1000, 4000 m
- c_{iw} = 0.5, 1, 1.5 m/s
- W = 4, 7, 11, 15 m/s

Simulations for C-band with fixed incidence angle of 30 degrees are plotted on Figure 3 for three different wind directions. We see that the simulated values indeed follow nicely the assumed relationship. Some more scattering and higher modulations are found for wind along the ISW propagation direction; this could be related to resonance between wind speed and the ISW group velocity.

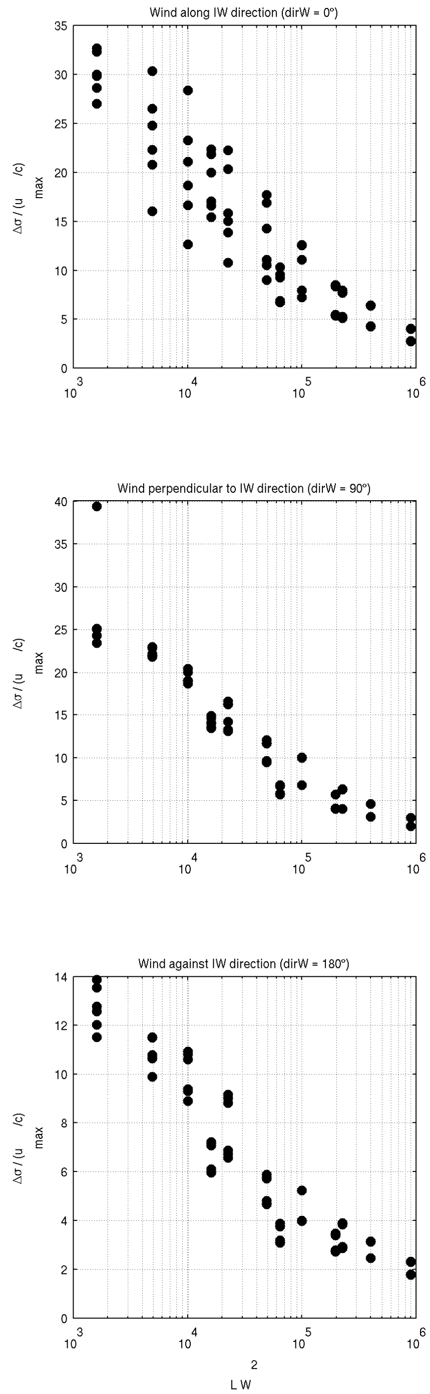

Figure 3: Relationship given by Eq. (12) simulated with the RIM model for VV polarization, 30 degrees incidence angle and wind along (top), perpendicular to (middle) and opposite to (bottom) the ISW propagation direction.

By extending the above exercise, α can be tabulated for any radar wavelength and wind direction. Care must however be taken for steep incidence angles, where the radar signal is dominated by specular reflection which may not follow the relation of Eq. 12.

4. CONCLUSIONS

A sophisticated radar imaging model is demonstrated to nicely reproduce observed modulation of the NRCS as induced by an internal solitary wave. Based on dimensional analysis, a simplified expression is derived, relating the NRCS modulation to geophysical parameters including the width and phase speed of the soliton, the strength of the induced surface current, and the ambient wind speed and direction. This simplified analytical expression (transfer function) is found to agree well with simulations with the full radar imaging model. Sometimes when the generation mechanism of the ISW is known to be the tide with known interval, the phase speed may be estimated from the SAR image by measuring the distance between two following solitons, or packets of solitons. In this case, or if other assumptions are made, the derived transfer function may be used to infer the relation between the pycnocline depth and strength, and the amplitude of the soliton. The derived methodology needs however careful validation by coincident in situ and radar measurements. The method may also be generalised to solitons of elevation, and regular internal waves.

ACKNOWLEDGEMENT
This work has been supported by the ESA-MOST Dragon programme. Envisat ASAR image is copyright ESA.

5. REFERENCES

1. Alpers, W., *Theory of radar imaging of Internal Waves,* Nature, 314, 245-247, 1985.
2. Johannessen, J. A., V. Kudryavtsev, D. Akimov, T. Eldevik, N. Winther, and B. Chapron, (2005), *On radar imaging of current features. Part 2: Mesoscale eddy and current front detection.* J.Geoph.Res.. 110, C07017.
3. Kudryavtsev, V., Hauser, D., Caudal, G., and Chapron, B. (2003a). *A semiempirical model of the normalized radar cross-section of the sea surface 1. Background model*. Journal of Geophysical Research (Oceans), 108:8054–+. doi: 10.1029/2001JC001003.
4. Kudryavtsev, V., Hauser, D., Caudal, G., and Chapron, B. (2003b). *A semiempirical model of the normalized radar cross section of the sea surface, 2. Radar modulation transfer function*. Journal of Geophysical Research (Oceans), 108:8055–+. doi:10.1029/2001JC001004.
5. Kudryavtsev, V., D. Akimov, J. Johannessen, and B. Chapron (2005), *On radar imaging of current features: 1. Model and comparison with observations, J.* Geophys. Res., 110, C07016, doi:10.1029/2004JC002505.
6. Korteweg, D.J. & de Vries, G (1895) On the change of long waves advancing in a rectangular canal and a new type of long stationary waves. Phil. Mag.. 5.